Math 45 SSM 2/e 6.6 Solving Polynomial Equations by Factoring

Objectives
1) Understand the zero product property
2) Use factoring and the zero product property to solve quadratic and polynomial equations.
3) Distribute and collect terms on one side, then solve quadratic and polynomial equations.

① Solve $\frac{5x}{5} = \frac{0}{5}$

$\boxed{x = 0}$

② If $x \cdot y = 0$, what do we know?

Either $\boxed{x=0}$ or $\boxed{y=0}$

IF x and y are factors of the product $x \cdot y = 0$
THEN Either the first factor x must be zero OR the second factor y must be zero

} This is called the zero-product property.

* CAUTION: MUST HAVE
- Expressions that are completely factored.

AND
- Expressions that are equal to zero

* "Completely factored" means ONE TERM.

Example: $x \cdot y = 2$

could be $x=1, y=2$
$x=2, y=1$
$x=\frac{1}{2}, y=4$
$x=3, y=\frac{2}{3}$
⋮
infinitely many combinations. ⇒ can't solve equation by saying $x=2$ and $y=2$.

③ Solve $(x+4)(2x-5)=0$

step 1: set expression = 0 using add + subtract
step 2: check it's factored completely (one term)
step 3: set each factor equal to zero:

$x+4=0$ $\qquad$ $2x-5=0$.

step 4: solve the resulting equations.

$\boxed{x=-4}$ $\qquad$ $2x=5$, $\boxed{x=5/2}$

MathXL will write the solutions as a solution set:

$\boxed{\{-4, \frac{5}{2}\}}$

I'll accept either the solution set or $\boxed{x=-4}$ and $\boxed{x=\frac{5}{2}}$.

optional step 5: check each answer by substituting into original equation.

$(x+4)(2x-5)=0$

$x=-4$ $(-4+4)(2\cdot(-4)-5)=0$

$(0)(-8-5)=0$

$(0)(-13)=0$

$0=0$ ✓ yes $x=-4$ is a solution

$x=\frac{5}{2}$ $(x+4)(2x-5)=0$

$(\frac{5}{2}+4)(2\cdot\frac{5}{2}-5)=0$

$(\frac{5}{2}+\frac{8}{2})(5-5)=0$

$(\frac{13}{2})(0)=0$

$0=0$ ✓ yes, $x=\frac{5}{2}$ is a solution.

(4) Solve $3x^2+5x=14x$

step 1: use add or subtract to set $=0$.

$3x^2+5x=14x$

$-14x \quad -14x$

$3x^2-9x=0$

step 2: Factor completely.

GCF $3x(x-3)=0$

step 3: Set each factor $=0$.

$3x=0$ $\quad x-3=0$

step 4: solve the resulting equations.

$\frac{3x}{3}=\frac{0}{3}$ $\quad x-3=0$

$\boxed{x=0}$ $\quad \boxed{x=3}$

or $\boxed{\{0,3\}}$

check:

$x=0$: $3(0)^2+5(0)=14(0)$

$0=0$ ✓

$x=3$: $3(3)^2+5(3)=14(3)$

$3\cdot 9+15=42$

$27+15=42$ ✓

⑤ Solve $(2x+5)(x-3) = 6x$

step1: Make expression = 0 using add or subtract.

$(2x+5)(x-3) - 6x = 0.$

step2: Factor completely. LHS is two terms! ☹
Must multiply $(2x+5)(x-3)$ and start factoring again.

$2x^2 - 6x + 5x - 15 - 6x = 0.$ FOIL

$2x^2 - 7x - 15 = 0$ combine

$2x^2 - 10x + 3x - 15 = 0$ double x

$2x(x-5) + 3(x-5)$

$(x-5)(2x+3) = 0$

step3: Set each factor equal to zero.

$x-5=0 \qquad 2x+3=0$

step4: Solve resulting equations

$\boxed{x=5}$ $\qquad 2x=-3$

$\boxed{x=-\frac{3}{2}}$

or $\boxed{\{5, -\frac{3}{2}\}}$

check:

$x=5$:

$(2\cdot 5+5)(5-3) = 6(5)$

$(10+5)(2) = 30$

$15 \cdot 2 = 30$ ✓

$x=-\frac{3}{2}$:

$\left(2\cdot\left(-\frac{3}{2}\right)+5\right)\left(-\frac{3}{2}-3\right) = 6\left(-\frac{3}{2}\right)$

$(-3+5)\left(-\frac{3}{2}-\frac{6}{2}\right) = -9$

$2\left(-\frac{9}{2}\right) = -9$ ✓

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⑥ $(w-3)^2 = 9+2w$

$(w-3)^2 - 9 - 2w = 0$ set = 0.

$(w-3)(w-3) - 9 - 2w = 0$

$w^2 - 6w + 9 - 9 - 2w = 0$ FOIL

$w^2 - 8w = 0$ combine

$w(w-8) = 0$ factor

$w=0 \quad w-8=0$ set factors = 0.

$\boxed{w=0}$ $\boxed{w=8}$ solve each equation

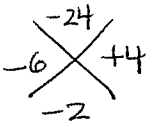
or $\boxed{\{0, 8\}}$

(7) $-3x^2 + 6x + 72 = 0$

$-3(x^2 - 2x - 24) = 0$

$-3(x-6)(x+4) = 0$

GCF with negative



$-3 \neq 0$ Contradiction

$x-6=0$ $\boxed{x=6}$

$x+4=0$ $\boxed{x=-4}$

or $\boxed{\{6, -4\}}$

Note: When the GCF has a coefficient $\neq 1$, we can divide all terms of both sides by that coefficient.

$$\frac{-3x^2}{-3} + \frac{6x}{-3} + \frac{72}{-3} = \frac{0}{-3}$$

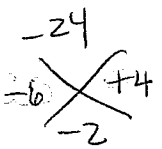
$x^2 - 2x - 24 = 0$

$(x-6)(x+4) = 0$

$x-6=0$ $\boxed{x=6}$

$x+4=0$ $\boxed{x=-4}$

or $\boxed{\{6, -4\}}$



(8) A ball is thrown vertically upward from the top of a 96-ft-tall building with initial velocity 80 ft per second. Solve the equation $-16t^2 + 80t + 96 = 192$ to find the time t, in seconds when the ball is 192 ft high.

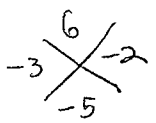
$-16t^2 + 80t + 96 = 192$

$-192 \quad -192$ subtract 192 to make equal to zero

$$\frac{-16t^2}{-16} + \frac{80t}{-16} - \frac{96}{-16} = \frac{0}{-16}$$ divide by GCF = -16.

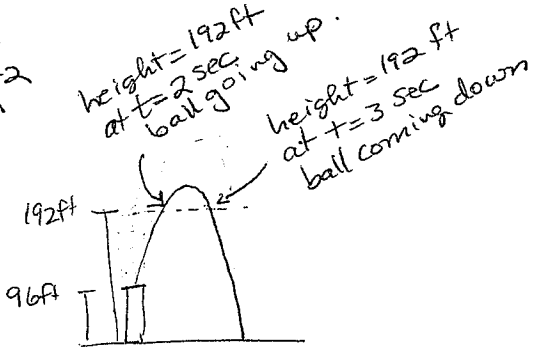
$t^2 - 5t + 6 = 0$

$(t-3)(t-2) = 0$



$t-3=0$ $\boxed{t=3 \text{ sec}}$

$t-2=0$ $\boxed{t=2 \text{ sec}}$



(9) $(a+3)(a-5)(3a+2)=0$

step 1: already $=0$.

step 2: already factored completely.

step 3: set factors $=0$.

$a+3=0 \quad a-5=0 \quad 3a+2=0$

step 4: solve each resulting equation

$\boxed{a=-3} \quad \boxed{a=5} \quad 3a=-2$

$\boxed{a=-\frac{2}{3}}$

or $\boxed{\left\{-3, 5, -\frac{2}{3}\right\}}$

(10) $2n^3+4=n^2+8n$

$-n^2-8n \quad -n^2 \quad -8n$ subtract

$2n^3-n^2-8n-4=0$ standard form

$n^2(2n-1)-4(2n-1)=0$ 4 terms $\Rightarrow$ use grouping

$(2n-1)(n^2-4)=0$

$(2n-1)(n+2)(n-2)=0$ difference of squares

$2n-1=0 \quad n+2=0 \quad n-2=0$ set factors $=0$.

$2n=1 \quad \boxed{n=-2} \quad \boxed{n=2}$ solve resulting equations

$\boxed{n=\frac{1}{2}}$

or $\boxed{\left\{\frac{1}{2}, -2, 2\right\}}$

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(14) $y^2+5y = 5(y+20)$

$0 = 5(y+20) - y^2 - 5y$ make equal to zero using add or subtract

$0 = 5y+100-y^2-5y$ distribute

$0 = -y^2+100$ standard form, combine

$0 = -(y^2-100)$ factor out GCF $= -1$.

$0 = -(y-10)(y+10)$ factor difference of squares.

set factors $= 0$

$y-10=0$ $\quad$ $y+10=0$

$-1=0$ (contradiction) $\quad$ $\boxed{y=10}$ $\quad$ $\boxed{y=-10}$

$\boxed{\{10,-10\}}$

Note #1: We could have moved everything to the LHS:

$y^2+5y-5(y+20)=0$

$y^2+5y-5y-100=0$

$y^2-100=0$

$(y-10)(y+10)=0$

$\boxed{y=10}$ $\quad$ $\boxed{y=-10}$, as before

Note #2: We could also have multiplied both sides of the equation by -1:

$0 = -y^2+100$

$0(-1) = (-y^2)(-1)+(100)(-1)$

$0 = y^2-100$

↓ continue the same as Note #1

(12) Solve $2a(a+1) = a^2+8$

$2a(a+1) - a^2 - 8 = 0$ make equal to zero using add or subtract

$2a^2 + 2a - a^2 - 8 = 0$ distribute

$a^2 + 2a - 8 = 0$ combine

$(a+4)(a-2) = 0$ factor completely

(-8; 4, -2; 2)

$a+4=0$ $\quad a-2=0$ set factors equal to zero and solve.

$\boxed{a=-4}$ $\quad \boxed{a=2}$

or $\boxed{\{-4, 2\}}$

(13) $(2k-3)(2k^2-9k-5) = 0$

continue factoring

$2k^2-9k-5$

$2k^2-10k+k-5$

$2k(k-5)+1(k-5)$

$(k-5)(2k+1)$

(-10, +1; -10, -9)

$(2k-3)(k-5)(2k+1) = 0$ original question now factored completely

$2k-3=0$ $\quad k-5=0$ $\quad 2k+1=0$

$2k=3$ $\quad 2k=-1$

$\boxed{k=\frac{3}{2}}$ $\quad \boxed{k=5}$ $\quad \boxed{k=-\frac{1}{2}}$

or $\boxed{\left\{\frac{3}{2}, 5, -\frac{1}{2}\right\}}$

(14) $z^2 + \frac{29}{4}z = 6$

$z^2 + \frac{29}{4}z - 6 = 0$ subtract

$4 \cdot z^2 + 4 \cdot \frac{29}{4}z - 4 \cdot 6 = 4 \cdot 0$ mult by 4 to clear fractions

$4z^2 + 29z - 24 = 0$

$4z^2 - 3z + 32z - 24 = 0$

$z(4z-3) + 8(4z-3) = 0$

(-96; -3, 32; +29)

-1, 96
-2, 48
-3, 32
-4, 24
-6, 16

(14) cont.

$(4z-3)(z+8)=0$

$4z-3=0 \qquad z+8=0$

$4z=3$

$\boxed{z=\frac{3}{4}} \qquad \boxed{z=-8}$

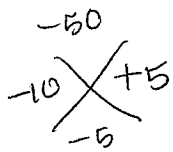
or $\boxed{\left\{\frac{3}{4}, -8\right\}}$

(15) $(x-2)(x-3)=56$

$(x-2)(x-3)-56=0$

$x^2-3x-2x+6-56=0$

$x^2-5x-50=0$



$(x-10)(x+5)=0$

$x-10=0 \qquad x+5=0$

$\boxed{x=10} \qquad \boxed{x=-5}$

or $\boxed{\{10, -5\}}$

(16) $-2x+3=3x^2-2x^3$

$2x^3-3x^2-2x+3=0$ make equal to zero

$x^2(2x-3)-1(2x-3)=0$ } grouping

$(2x-3)(x^2-1)=0$

$(2x-3)(x+1)(x-1)=0$ difference of squares

$2x-3=0 \qquad x+1=0 \qquad x-1=0$

$2x=3$

$\boxed{x=\frac{3}{2}} \qquad \boxed{x=-1} \qquad \boxed{x=1}$

or $\boxed{\left\{\frac{3}{2}, -1, 1\right\}}$

(17) $\frac{18x^3}{3} + \frac{3x^2}{3} - \frac{6x}{3} = \frac{0}{3}$

$6x^3 + x^2 - 2x = 0$

$x(6x^2 + x - 2) = 0$ GCF

-12, 4, -3, 1

$x[6x^2 + 4x - 3x - 2] = 0$

$x[2x(3x+2) - 1(3x+2)] = 0$

$x(3x+2)(2x-1) = 0$

$x = 0$ $\quad 3x+2=0$ $\quad 2x-1=0$

$3x = -2$ $\quad 2x = 1$

$x = -\frac{2}{3}$ $\quad x = \frac{1}{2}$

$\{0, -\frac{2}{3}, \frac{1}{2}\}$

Extras

(18) $3m^2 - 3m = 2 - 2m$

(19) $4k^2 + 9 = -12k$

(20) $9q^2 = 3q + 6$

(21) $7y + 3 = 2y - 12$

(22) $p^2 - p - 20 = 0$

(23) Find 3 consecutive odd integers such that the product of the second and third is 99.

(24) The length and width of a garden are consecutive even integers and the area is 440 sq. feet. Find the dimensions.

(25) $3x^3 + x^2 - 14x = 0$

(26) $m^3 + 2m^2 - 9m - 18 = 0$

(27) $-2x + 3 = 3x^2 - 2x^3$

(28) $(2a-1)^2 = 16$

(29) $4p - 3 = -4p^2$

(30) $b^2 + 18 = 11b$

(31) $2x^2 = 5x$

(32) $a^2 - 6a + 9 = 0$

(33) $14x - 49x^2 = 0$

(34) $n^2 + 9n + 14 = 0$.